DETERMINATION OF FORCE BY STIFFNESS-MATRIX FOR 3-D MODEL USED IN ROBOTIC END-EFFECTOR

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ABSTRACT

In this Paper, a basic topic about force of 3-D model is determined by stiffness-matrix. This model consists of six legs attached with two different diameter platforms is shown. These legs, having springs inside, are combined with prismatic in middle and by spherical joints in ends with platforms. The stiffnesses of these were checked in the laboratory. Through these spring-stiffnesses, displacement of each leg was calculated to determine the concerned applied force. Then, these local and global displacements were calculated theoretically, by using Jacobian matrix and wrench analysis. This Jacobian matrix is calculatd by kinematics analysis. These experimental and theoretical results are compared. It is a new research to determine the force of a new 3-D model.

1. INTRODUCTION

In an elastic system where a rigid body is grasped by a compliant mechanism, the stiffness is defined as a linear map from an infinitesimal displacement space to the infinitesimal wrench space, in an equilibrium condition.

Considering a stiffness kI along the ith direction of a contact, all stiffnesses can be assembled into a stiffness matrix [k], which contains the stiffness of contacts in its diagonal elements. The stiffnesses can further be mapped into the geometry of grasp, by applying the congruence transformation used to form a grasp stiffness matrix [1].

A preload can be applied in two different ways. The first will be applied parallel to the contact-normal Such preload not only affects the contact-force at each contact-normal, but also lends itself directly to supposition of forces [2-4] The second will be used to apply a preload through the contact normals.

The contact stiffness is the key in this study. The study of stiffness in the domain of screw-theory can be dated back to 1965, when a stiffness-matrix was introduced by Dimentberg [5] for static and dynamic loading conditions. The stiffness and compliance were further analyzed by Loncaric [6] with the help of linear algebra. Further, a contact model with preloaded springs was introduced and used for both frictionless and frictional-grasp analysis and synthesis [2-4].

Meanwhile, a loaded three-line spring, system for synthesis of stiffness was proposed [1], wherein it was demonstrated that general spatial stiffness can be modeled with Stewart platform-type parallel mechanism, consisting of passive line springs.

2. METHODOLOGY

A 3-D model possesses six legs, consisting of two parts, 1-male and 2-female (In which springs are inserted) which are combined by prismatic joints in middle and spherical joints with two different diameter platforms as shown in Figure-1. The local displacements are noted in Table-1. Then stiffnesses of these springs were checked as in Table-2 on basis of these displacements. In Table-3, Global displacements are observed by central load.

2.1 Stiffness-Matrix

Stiffness is the property of material, through which its strength can be analyzed on application of force. Here we would calculate the stiffness of the spring. This stiffness is determined through displacement in compression on applying various forces Mathematically:

$F = K.\partial d$

Or $K = F/\partial d$, where

K = Stiffness of Spring

F = Force

∂d_1 to ∂d_6 = displcements in compression

f_1 to f_6 = Local forces

∂D = Global displacement

k_1 to k_6 = Stiffneses

In Matrix form

$$\begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \\ f6 \end{bmatrix} = \begin{bmatrix} k1 & 0 & 0 & 0 & 0 & 0 \\ 0 & k2 & 0 & 0 & 0 & 0 \\ 0 & 0 & k3 & 0 & 0 & 0 \\ 0 & 0 & 0 & k4 & 0 & 0 \\ 0 & 0 & 0 & 0 & k5 & 0 \\ 0 & 0 & 0 & 0 & 0 & k6 \end{bmatrix} . \begin{bmatrix} \partial d1 \\ \partial d2 \\ \partial d3 \\ \partial d4 \\ \partial d5 \\ \partial d6 \end{bmatrix}$$

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Table - 1: Spring Deflection Under Application of Force

S. NO	Load (F) In N	Local Displacement (δd) in compression for legs in mm																	
		δd1			δd2			δd3			δd4			δd5			δd6		
		δdi	δdf	Av	δdi	δdf	Av	δdi	δdf	Av	δdi	δdf	Av	δdi	δdf	Av	δdi	δdf	Av
1	9.81	0.95	1.2	1.075	0.75	1.10	0.925	0.85	1.01	0.926	1.0	1.15	1.075	1.02	1.20	1.1	1.0	1.20	1.10
2	2.943	1.50	2.0	1.75	1.70	2.01	1.85	1.80	2.0	1.90	1.75	2.10	1.925	1.80	2.10	1.95	1.9	2.10	2.0
3	4.905	3.0	3.0	3.0	2.8	2.8	2.8	2.90	2.90	2.90	3.1	3.10	3.10	3.10	3.10	3.10	3.0	3.0	3.0

Table-2: Stiffness of Spring Based upon Above Readings

S. No.	Stiffness of Spring (K = F δd) for each leg in N/mm						Average Stiffness (K_{av}) in Newton (N)					
	K1	K2	K3	K4	K5	K6	K_{av1}	K_{av2}	K_{av3}	K_{av4}	K_{av5}	K_{av6}
1	0.9126	1.0605	1.0606	0.9126	0.8926	0.8920	1.4098	1.4677	1.4336	1.3412	1.3278	1.3328
2	1.6816	1.5908	1.5489	1.5288	1.5092	1.4727						
3	1.6349	1.7513	1.6913	1.5823	1.5823	1.6351						
Total	4.2291	4.4031	4.3008	4.0236	3.9834	3.9932						

Table-3: Global Displacements

Gen; Load Kept centrally (W)	Local Displacement in compression for legs in mm (δd)																	
	δd1			δd2			δd3			δd4			δd5			δd6		
	δdi	δdf	dif	δdi	δdf	dif	δdi	δdf	dif	δdi	δdf	dif	δdi	δdf	dif	δdi	δdf	dif
-9.81N	8.0	-9.5	-1.5	6.9	-8.4	-1.5	7.0	-8.3	-1.3	7.8	-9.2	-1.4	7.0	-8.3	-1.3	9.0	-10.2	-1.2

3. RESULTS AND DISCUSSION:

Here we discuss the results of the practical and kinematics analysis performed in the previous section. Here Jacobian is achieved by kinematics analysis. The local displacement is gained by individual leg, while global is taken centrally as reference. There are bound to be some errors and differences in the theoretical and practical values. This may be due to incorrect instrument and friction play in the designed model. The reasons for justification of errors have been discussed in the following sections.

3.1 Investigation of Stiffness Mappings

Stiffness mapping can be achieved with the following two different steps.

a) Experimental Setup

The model consists of springs in its six legs. Every spring was tested in the laboratory for determining its stiffness. These springs are used for compression force, when the force is applied on each leg of the model, it gives different values of displacements along each of six legs of the model. However, three different readings were noted by application of different loads on each spring for the stiffness. This has been given in Table-1. Therefrom the average stiffness value of each leg was calculated. These stiffnesses were set along the diagonal of a 6 matrix as shown in Table-2.

Similarly, applying the load to the designed model will produce displacements in each leg. The relationship of the resulting displacement can be

Since we know that $F = [K\}.\partial d_l$ Or $J_g \times F = J_g \times [K\}.\partial d_l$

i - e $\partial w = J_g.[k].J^T.\partial D_g$

where

$$\partial w = \begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ Mz \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -9.81 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and '$J_g.[k].J^T.$' is called congruence transformation denoted by (K_g)

$$\begin{bmatrix} 0 \\ 0 \\ -9.81 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.4330 & 0.0000 & -0.5000 & -0.4330 & -0.0000 & 0.5000 \\ 0.2500 & 0.5000 & 0.0000 & -0.2500 & -0.5000 & -0.0000 \\ 0.8660 & 0.8660 & 0.8660 & 0.8660 & 0.8660 & 0.8660 \\ -48.118 & -96.237 & -48.118 & 48.118 & 96.2371 & 48.118 \\ 83.343 & 0 & -83.343 & -83.343 & 0 & 83.343 \\ 0.0000 & 0.0000 & -27.781 & 0.0000 & 0.0000 & -27.781 \end{bmatrix} . \begin{bmatrix} 1.4098 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.4677 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.4335 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6021 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.3278 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.3328 \end{bmatrix}$$

$$\begin{bmatrix} 0.4330 & 0.2500 & 0.8660 & -48.1185 & 83.3438 & 0.0000 \\ 0.0000 & 0.5000 & 0.8660 & -96.2371 & 0 & 0.0000 \\ -0.5000 & 0.0000 & 0.8660 & -48.1185 & -83.3438 & -27.781 \\ -0.4330 & -0.2500 & 0.8660 & 48.1185 & -83.3438 & 0.0000 \\ -0.0000 & -0.5000 & 0.8660 & 96.2371 & 0 & 0.0000 \\ 0.5000 & -0.0000 & 0.8660 & 48.1185 & 83.3438 & -27.781 \end{bmatrix} . \begin{bmatrix} \partial D_{g1} \\ \partial D_{g2} \\ \partial D_{g3} \\ \partial D_{g4} \\ \partial D_{g5} \\ \partial D_{g6} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -9.81 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 1.0e+004 \times \begin{bmatrix} 0.0002 & 0.0001 & -0.0001 & -0.0038 & 0.0296 & 0.0001 \\ 0.0001 & 0.0001 & -0.0000 & -0.0195 & 0.0104 & -0.0000 \\ -0.0001 & -0.0000 & 0.0008 & 0.0075 & -0.0166 & -0.0067 \\ -0.0038 & -0.0195 & 0.0075 & 4.3900 & -0.9006 & 0.0135 \\ 0.0296 & 0.0104 & -0.0166 & -0.9006 & 5.4029 & 0.0233 \\ 0.0001 & -0.0000 & -0.0067 & 0.0135 & 0.0233 & 0.2135 \end{bmatrix} . \begin{bmatrix} \partial D_{g1} \\ \partial D_{g2} \\ \partial D_{g3} \\ \partial D_{g4} \\ \partial D_{g5} \\ \partial D_{g6} \end{bmatrix}$$

$$\begin{bmatrix} \partial D_{g1} \\ \partial D_{g2} \\ \partial D_{g3} \\ \partial D_{g4} \\ \partial D_{g5} \\ \partial D_{g6} \end{bmatrix} = 1.0e+004 \times \begin{bmatrix} 0.0002 & 0.0001 & -0.0001 & -0.0038 & 0.0296 & 0.0001 \\ 0.0001 & 0.0001 & -0.0000 & -0.0195 & 0.0104 & -0.0000 \\ -0.0001 & -0.0000 & 0.0008 & 0.0075 & -0.0166 & -0.0067 \\ -0.0038 & -0.0195 & 0.0075 & 4.3900 & -0.9006 & 0.0135 \\ 0.0296 & 0.0104 & -0.0166 & -0.9006 & 5.4029 & 0.0233 \\ 0.0001 & -0.0000 & -0.0067 & 0.0135 & 0.0233 & 0.2135 \end{bmatrix}^{-1} . \begin{bmatrix} 0 \\ 0 \\ -9.81 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \partial D_{g1} \\ \partial D_{g2} \\ \partial D_{g3} \\ \partial D_{g4} \\ \partial D_{g5} \\ \partial D_{g6} \end{bmatrix} = \begin{bmatrix} -0.6131 \\ 3.3722 \\ -1.9119 \\ 0.0165 \\ -0.0048 \\ -0.0596 \end{bmatrix}$$

But we also know from above equation that $\partial d_l = [J_g^T].\partial D_g$

$$\begin{bmatrix} \partial d_{l1} \\ \partial d_{l2} \\ \partial d_{l3} \\ \partial d_{l4} \\ \partial d_{l5} \\ \partial d_{l6} \end{bmatrix} = \begin{bmatrix} 0.4330 & 0.2500 & 0.8660 & -48.1185 & 83.3438 & 0.0000 \\ 0.0000 & 0.5000 & 0.8660 & -96.2371 & 0 & 0.0000 \\ -0.5000 & 0.0000 & 0.8660 & -48.1185 & -83.3438 & -27.7812 \\ -0.4330 & -0.2500 & 0.8660 & 48.1185 & -83.3438 & 0.0000 \\ -0.0000 & -0.5000 & 0.8660 & 96.2371 & 0 & 0.0000 \\ 0.5000 & -0.0000 & 0.8660 & 48.1185 & 83.3438 & -27.7812 \end{bmatrix} . \begin{bmatrix} -0.6131 \\ 3.0656 \\ -1.9119 \\ 0.0251 \\ -0.0144 \\ -0.0948 \end{bmatrix} = \begin{bmatrix} -2.2697 \\ -1.5543 \\ -0.0865 \\ -1.0418 \\ -1.7572 \\ 0.0865 \end{bmatrix} \text{mm}$$

From above

From above calculations we get following equation to solve

$$F = K . \partial d_1$$

$$\begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{bmatrix} = \begin{bmatrix} 1.4098 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.4677 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.4335 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6021 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.3278 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.3328 \end{bmatrix} . \begin{bmatrix} -2.2697 \\ -1.5543 \\ -0.0865 \\ -1.0418 \\ -1.7572 \\ 0.0865 \end{bmatrix}$$

$$\begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{bmatrix} = \begin{bmatrix} -3.1998 \\ -2.2812 \\ -0.1240 \\ -3.7527 \\ -2.3332 \\ 0.1153 \end{bmatrix} \quad N$$

These values of the forces are calculated based upon theoretically local displacements through coordinate system.

Also Similiarily putting the experimental values of local displacements in equation $[F] = [K].[\partial d_l]$

$$\begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{bmatrix} = \begin{bmatrix} 1.4098 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.4677 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.4335 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6021 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.3278 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.3328 \end{bmatrix} . \begin{bmatrix} -1.5 \\ -1.5 \\ -1.3 \\ -1.4 \\ -1.3 \\ -1.2 \end{bmatrix}$$

$$\begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{bmatrix} = \begin{bmatrix} -2.1146 \\ -2.2016 \\ -1.8636 \\ -5.0430 \\ -1.7261 \\ -1.5993 \end{bmatrix} \quad N$$

'Negative Sign' in force shows that direction of applied force is downward. The values of the forces were calculated, based upon practically local displacements.

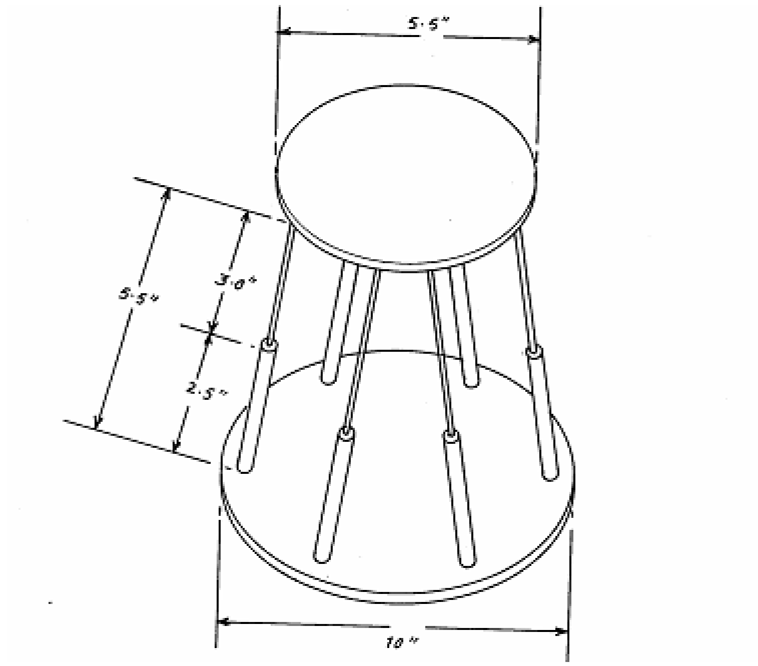

Figure - 1: Elevation View of Proposed 3-D Model

interpreted into global coordinates, using the relationship.

$D_g = [J].d_l$

As we apply the wrench 'W', it gives d and then, using transformation relationship,

$W = K_g\ D_g$

b) Theoretical analysis

Any wrench-vector in its space under the mapping is a linear combination of these six wrench-vectors. Once the wrench-space is completely defined, the stiffness matrix for the mapping can be specified.

Since we know that
$F = [K].d_l$

multiplying both sides by [J] we get
$[J].\ F = [J].\ [K].d_l$

Or $\quad W = [J].\ [K].d_1$

But $\quad d = [J^t].\ D$
Hence $W = [J].\ [K].\ [J^t].D_g$

These equations are used in the above section of methodology. Where D and d are global and local displacements, while [K], [J] and W are stiffness matrix, Jacobian matrix and wrench analysis respectively. The D or d then can be compared with that practically obtained.

4. CONCLUSION

These determined stiffnesses were used for practical and theoretical analysis. For practically stiffness mapping, it gives the required local displacements depending on its applied loads.

For theoretical purposes, when same stiffness-mapping is applied for determining the local displacements through Jacobian and congruence transformation, it gives different local displacements. There are a no of reasons for the un-matching results, which are given below:

a. Design of Model: The model has not been designed with sophisticated machines, rather is manufactured at workshop level. The aim was concept approval. It has a number of joints, which have a lot of play. These plays affect the readings along the each leg and are a major source of error. It can be eliminated by careful and accurate fabrication.

b. Measurement of Parameters: The displacement along each leg is measured through the readings on the scale drawn on each leg. Again, it is not of high resolution. The thickness of the lines, the least count and parallaxes are the major sources of errors. These can be improved by incorporating the digital kind of instrumentation.

c. Application of load: The load has been considered to be applied along the global axes of the model, which are along the center of the bottom plate. Specifically, it has been applied along z-axis, placing upon upper plate. Ideally and theoretically, it should be in the center and parallel to global z-axis. Placement of the center is associated with some errors, as it was approximate one. A better arrangement, scale marking, and small loads would have increased the accuracy of the results.

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